of the base mid way between the upper and lower bases ; and a=altitude of frustum. Then $\rho=\frac{1}{2}(R+r)$. $\therefore 4\rho^2=(R+r)^2$.

Volume of frustum$=\frac{1}{3}\pi a(R^2+r^2+Rr)=\frac{1}{6}\pi a(2R^2+2r^2+2Rr)=\frac{1}{6}\pi a[R^2+r^2+(R+r)^2]=\frac{1}{6}\pi a(R^2+r^2+4\rho^2)$.

The same method applies to frustums of pyramids, and all solids coming under the prismatoid formula as special cases.

Also solved in a more general manner by *B. F. SINE*, *P. S. BERG*, *J. SCHEFFER*, *HARVEY N. DAVIS*, *W. H. DRANE*, and *CHAS. C. CROSS*.

CALCULUS.

68. Proposed by EDWARD DRAKE ROE, JR., A. M., Associate Professor of Mathematics, Oberlin College, Oberlin, Ohio.

If $a^{x^{\cdot^{\cdot^{x}}}}$ to r steps be denoted by $a^{\overset{r}{x}}$, and if $y=x^{\overset{r}{x}}$, prove that

$$D_xy=x^{\overset{r}{x}+\overset{r-1}{x}+\dots+\overset{1}{x}}(\log x)^{r-1}(1+\log x)+\sum_{k=2}^{k=r}x^{\overset{r}{x}+\overset{r-1}{x}+\dots+\overset{k-1}{x}-1}(\log x)^{r-k}.$$

Solution by the PROPOSER.

If $y=f_1(x)^{f_2(x)}$, we obtain, by taking the logarithm of both sides of the equation, and differentiating, the formula

$$D_xy=f_1(x)^{f_2(x)}\log f_1(x)D_xf_2(x)+f_1(x)^{f_2(x)-1}f_2(x)D_xf_1(x).$$

If in this $f_1(x)=x$, $f_2(x)=x^x$, that is if $y=x^{x^x}=x^{\overset{2}{x}}$, we obtain

$$D_xy=x^{\overset{2}{x}+\overset{1}{x}}\log x(1+\log x)+x^{\overset{2}{x}+\overset{1}{x}-1},$$

and the formula to be proved is true when $r=2$. It is evidently not true for values of $r<2$. Assume that it is true for all other values of r

Let $y=x^{\overset{r+1}{x}}=x^{x^{\overset{r}{x}}}$. In the above formula put, $f_1(x)=x$, $f_2(x)=x^{\overset{r}{x}}$, and we get

$$D_xy=x^{\overset{r+1}{x}}\log xD_xx^{\overset{r}{x}}+x^{\overset{r+1}{x}-1}x^{\overset{r}{x}},$$

but by this assumption this is

$$D_xy=x^{\overset{r+1}{x}}\log x[x^{\overset{r}{x}+\overset{r-1}{x}+\dots+\overset{1}{x}}(\log x)^{r-1}(1+\log x)]+x^{\overset{r+1}{x}}\log x\sum_{k=2}^{k=r}x^{\overset{r}{x}+\overset{r-1}{x}+\dots+\overset{k-1}{x}-1}(\log x)^{r-k}$$
$$+x^{\overset{r+1}{x}+\overset{r}{x}-1}$$
$$=x^{\overset{r+1}{x}+\overset{r}{x}+\overset{r-1}{x}+\dots+\overset{1}{x}}(\log x)^{r}(1+\log x)+\sum_{k=2}^{k=r+1}x^{\overset{r+1}{x}+\overset{r}{x}+\dots+\overset{k-1}{x}-1}(\log x)^{r+1-k}.$$

But this expression has the same form with respect to $r+1$, that the assumption had with respect to r, and since the assumption was true for $r=2$, it is also true for all values of r greater than 2, which is what we had to prove.

Erlangen, Bayern, Hauptstrasse 83II, 26 February, 1898.

Also solved by *C. W. M. BLACK*, *W. W. LANDIS*, and *G. B. M. ZERR*.